Introduction to the various connectivity analyses: Phase-based and Power-based connectivity

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Types of Connectivity

- anatomical/structural:
  = presence of axonal connections

- functional:
  = correlation or coherence between nodes

- effective:
  = causal (directed) influences between neurons or neuronal populations

Park & Friston, 2013
Types of Connectivity

- Phase-based connectivity
  - Intersite phase clustering
  - Cross-spectral coherence
  - Phase-lag index

- Power-based connectivity

Park & Friston, 2013
Types of Connectivity

- Granger prediction
- Mutual information

Park & Friston, 2013
1. Phased-Based Connectivity
Recap: From Real to Complex

\[ u(t) \in \mathbb{R} \rightarrow_{FT} z(t) = H[u](t) \]

\[ = x + iy \]

\[ = A (\cos \varphi + i \sin \varphi) = Ae^{i\varphi} \]
Recap: Inter-trial phase clustering (ITPC)

\[ ISPC_f = \left| n^{-1} \sum_{t=1}^{n} e^{i(\varphi_{xt})} \right| \]

Filtered signal at 5 Hz

Mean vector length: 0.86603

Mean vector length: 0.70711
Intersite phase clustering (ISPC) difference

\[ ISPC_f = \left| n^{-1} \sum_{t=1}^{n} e^{i(\varphi_{xt} - \varphi_{yt})} \right| \]
ISPC over time

ISPC does not depend on the phase values themselves, but their consistency.
ISPC over time

- ISPC is computed in sliding time segments
- ISPC is non-directional
  - ISPC A->B = ISPC B->A

\[
ISPC_{f}^{Time}(T) = \left| \frac{1}{N_t} \sum_{t=1}^{N_t} e^{i(\phi_{xT} - \phi_{yT})} \right|
\]
SNR for Frequency & Temporal Precision Trade-offs

- **Trial-wise measure:** within each time segment, ISPC trial, we average across trials

- Trade-off between SNR and temporal precision in time segment length
  - Long segment mean better SNR but worse temporal precision
Intersite phase clustering (ISPC) over trials

\[ ISPC_{Trials}^{f}(t) = \frac{1}{N_T} \sum_{T=1}^{N_T} e^{i(\phi_{xtT} - \phi_{ytT})} \]

– Taking the *average* of phase angle differences between electrodes over time-points

t = time-points, \( \phi \) = phase angles, f = frequency, T = trials
ISPC over time vs. ISPC over trials

$$\langle ISPC_f \rangle_{TimeWindow} = N_t^{-1} \sum_{t=1}^{N_t} ISPC_f(t)$$

$$\langle ISPC_f \rangle_{Condition} = N_T^{-1} \sum_{T=1}^{N_T} ISPC_f(T)$$
ISPC over time vs. ISPC over trials

- **Time**
  - Less sensitive to temporal jitters
  - Measures *non-phase locked* and phase locked connectivity
  - Needs same temporal resolution between method and original data

- **Trials**
  - Stronger evidence for task-related modulation in connectivity
  - No extra loss of temporal precision
  - Cannot be performed on resting-state data
Spectral Coherence

- Phase values weighted by \textit{power}

\[ Coher_{xy} = \frac{\left| \frac{1}{N_t} \sum_{t=1}^{N_t} m_{tx} \cdot m_{ty} e^{i(\phi_{ty} - \phi_{tx})} \right|^2}{\left( \frac{1}{N_t} \sum_{t=1}^{N_t} m_{tx}^2 \right) \cdot \left( \frac{1}{N_t} \sum_{t=1}^{N_t} m_{ty}^2 \right)} \]

\[ x = (x_1, \ldots, x_t, \ldots, x_{N_t})^T \]

\[ x_t = m_{tx} e^{i\phi_{tx}} \]
Spectral Coherence

- Relation to real-valued correlation (of mean-centered random variables $x$ and $y$):

$$Coher_{xy} = \left| \frac{\sum_{t=1}^{N_t} m_{tx} \cdot m_{ty} e^{i(\varphi_{ty} - \varphi_{tx})}}{||x||^2 \cdot ||y||^2} \right|^2$$

$$Coher_{xy} = \frac{|x^H y|^2}{||x||^2 \cdot ||y||^2} = \frac{|x^H y|^2}{x^H x \cdot y^H y} = |Corr_{xy}|^2 \in [0 \ldots 1]$$

$$x_t^* = m_{tx} e^{-i\varphi_{tx}}$$

$$x^H y = \sum_{t=1}^{N_t} x_t^* \cdot y_t$$
Complex Conjugate

$e^{i\varphi} = \cos \varphi + i \sin \varphi$

$A (\cos \varphi + i \sin \varphi) = Ae^{i\varphi}$

$A (\cos \varphi - i \sin \varphi) = A\bar{e}^{i\varphi}$
Spectral Coherence

- Phase values weighted by power

\[ Coher_{xy} = \left| \frac{1}{N_t} \sum_{t=1}^{N_t} m_{tx} \cdot m_{ty} e^{i(\varphi_{ty} - \varphi_{tx})} \right|^2 \]

- Normalizes, but issues arise if phase space is associated with increased/decreased power: Example!
Phase Lag-Based Measures

- Phase lag of zero or pi could indicate electrodes recording same source
  - volume conduction confounds
Phase Lag-Based Measures

- **Imaginary coherence** – spectral coherence that ignores volume conduction (only imaginary part)

- **Phase-lag index** – non-volume conducted connectivity produces positive or negative (relative to the imaginary axis) vectors
  - Less sensitive to amount of clustering
2. Power-Based Connectivity
Power-based Connectivity

- Correlating time-frequency power between two electrodes across **time** or **over trials**

- Does not assume connectivity is instantaneous, or at the same frequency

- Flexibility (wrt. experimental design)
Bivariate correlation coefficients
Pearson vs Spearman

- **Pearson correlation coefficient**: covariance of two variables, scaled by the variance of each variable

\[
 r = \frac{\sum_{t=1}^{n} (x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^{n} (x_t - \bar{x})^2 \sum_{t=1}^{n} (y_t - \bar{y})^2}}
\]

- **Assumption**: Data are normally distributed
Bivariate correlation coefficients
Pearson vs Spearman

- Rank-order data first

Spearman

\[ r = \frac{\sum_{t=1}^{n} (x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^{n} (x_t - \bar{x})^2 \sum_{t=1}^{n} (y_t - \bar{y})^2}} \]
Bivariate correlation coefficients
Pearson vs Spearman

- EEG data are non-normally distributed
- Presence of outliers
Power Correlations over Time

1) Pick 2 electrodes or sources
2) Compute power time series
3) Compute a correlation coefficient between time-varying power
Closest Link to fMRI: Functional Connectivity at “Rest”

Fox et al., 2005
1) T/f windows prior to analysis
   → Hypothesis driven (pre-defined windows)

2) At each time point over trials
   → Hypothesis driven (focus on two electrodes/sources and frequency bands)

3) “Seed” analysis
   → Exploratory
T/f windows prior to analysis

1. Select t/f windows for two electrodes
2. Extract power from that window for each trial (averaging over all points within time window)
3. Compute a single correlation coefficient
At each time point over trials

1. Gives you times series of correlation coefficients
2. Can use same (e.g. below) or different frequency bands
3. Lets you assess changes in connectivity over time
“Seed” analysis

- **Exploratory**
- **How:** Select a “seed” electrode (or source) and correlate the power time-series with cross-trial power in all other t/f points at one, some or all electrodes
Partial Correlation

A) Connectivity: OK

C) Connectivity: NOT OK

\[ r_{xy,z} = \frac{r_{xy} - r_{xz}r_{yz}}{\sqrt{1-r_{xz}^2} \sqrt{1-r_{yz}^2}} \]

Correlation between fz and f6

Partial correlation between fz and f6
Pros & Cons

- Depends on your research question
  - Strong hypotheses: ISPC with checks for volume conduction confounds
  - Exploratory: use phase-lag index or imaginary coherence or seed-based analysis

- Connectivity over trials vs time
  - Time: More sensitive to detecting coherence at high frequencies but poorer temporal resolution
  - Trial: More sensitive to transient changes
Thank You